Published by Institute of Physics Publishing for SISSA/ISAS
Received: *February 20, 2004*
Revised: *April 27, 2004*
Accepted: *May 4, 2004*

The masses of weakly-coupled scalar fields in the early Universe

David H. Lyth
Physics Department, Lancaster University
Lancaster LA1 4YB, U.K.
E-mail: d.lyth@lancaster.ac.uk

Takeo Moroi
Department of Physics, Tohoku University
Sendai 980-8578, Japan
E-mail: moroi@tuhep.phys.tohoku.ac.jp

Abstract: We consider the effective mass-squared in the early Universe, of a scalar field which has only Planck-suppressed couplings with light fields and whose true mass is less than the Hubble parameter H. A detailed investigation shows that the effective mass-squared generically is of order $\pm H^2$ during inflation and matter-domination, but much smaller during radiation domination. We consider the special circumstances under which the mass-squared may be much bigger or much smaller than the generic value.

Keywords: Cosmology of Theories beyond the SM, Physics of the Early Universe.

JHEP05(2004)004

Contents

1. Introduction

In the early Universe, the effective mass-squared of a scalar field ϕ will not necessarily be equal to its value in the vacuum (the true value). This is because the lagrangian will contain an interaction term of the form $f\phi^2$, where f is a function of the fields in the effective field theory. In the vacuum, the mass-squared is

$$m_\phi^2 = m_{\phi 0}^2 + \langle f \rangle \,, \tag{1.1}$$

where the first term comes from any mass term in the lagrangian and the second term represents the contributions from vacuum expectation values of fields. In the early Universe though, the expectation value $\langle f \rangle_t$ will differ from the vacuum value, and so will the value of the effective mass;

$$m_{\rm eff}^2 = m_{\phi 0}^2 + \langle f \rangle_t \,. \tag{1.2}$$

In particular, if the expectation value is much bigger than m_ϕ^2 we shall have simply

$$m_{\rm eff}^2 \simeq \langle f \rangle_t \,. \tag{1.3}$$

The expectation value in the early Universe may arise in a variety of ways. There may be time-independent or slowly varying scalar fields which are far from their vacuum values, and there may also be rapidly oscillating scalar fields. (Here 'slow' and 'rapid' refer to the timescale used to evaluate the expectation value, which is here a time-average. To make sense, the timescale should be at least the inverse of the effective mass.) After re- or pre-heating there is certainly a gas, described by particles of spin zero or higher, all of which contribute to the (now quantum) expectation value of f.

If ϕ has renormalizable interactions *and* ϕ is close to zero, these interactions will give the dominant contribution to $\langle f \rangle$. In particular, if the interactions place ϕ in thermal equilibrium with fields whose effective mass is much smaller than the temperature T, and m_ϕ is also much smaller, then typically [1]

$$m_{\rm eff}^2 \sim T^2 \,. \tag{1.4}$$

If instead ϕ is large, any field with which it has a renormalizable interactions will usually acquire a large mass-squared, which will suppress its contribution to $\langle f \rangle$. For example, if there is an interaction

$$V \supset \lambda \psi^2 \phi^2 \tag{1.5}$$

with a scalar field ψ, a large value of ϕ will generate a large effective mass-squared for ψ (equal to $\lambda\phi^2$) which will drive ψ to zero and eliminate the contribution of this interaction to the effective mass-squared of ϕ.

In this paper we suppose that only non-renormalizable interactions are significant in determining the effective mass-squared of the field ϕ, and that these interactions are suppressed by the Planck scale $M_{\rm P}$.[1] In that case the standard estimate [4, 5, 6] is that in the regime $H \gg m_\phi$

$$m_{\rm eff}^2 \sim \pm H^2 \,, \tag{1.6}$$

where $H(t)$ is the Hubble parameter and the sign may be time-dependent.

If present, this 'mass of order H' will be crucial for any field which plays a significant role in the early Universe. For instance, it will be crucial if ϕ is a field breaking Peccei-Quinn (PQ) symmetry [7], or a field responsible for Affleck-Dine baryogenesis [8] or a field which reheats the Universe [6, 9]. It will also be crucial if ϕ is a field responsible for the primordial density perturbation, whether the inflaton [10, 11] or some 'curvaton' field [12, 13, 14].

Actually, if ϕ is indeed the field responsible for the density perturbation, then its mass-squared m_*^2 during inflation must be at least a factor of 10 or so less than the rough estimate eq. (1.6). This is because of the prediction of the spectral index $n - 1 \simeq \frac{2}{3} V_*''/H_*^2$ (where V'' is the second derivative of the potential at horizon exit) together with the 1-σ estimate [15, 16] $n = 0.97 \pm 0.03$. Furthermore, if ϕ is not the inflaton then its mass-squared may need to be suppressed even after inflation [17] (until H falls to the true mass) so as to preserve the fractional perturbation in ϕ.

[1] We are not assuming that all non-renormalizable interactions are suppressed so strongly. For instance, in the Horava-Witten setup with higher-dimensional quantum gravity scale $\Lambda \sim 10^{-2} M_{\rm P}$, the non-renormalizable interactions of the effective four-dimensional field theory are in general suppressed by $\Lambda \ll M_{\rm P}$, leading [2] in general to an effective mass-squared of order $(M_{\rm P}/\Lambda)^2 H^2$. But if a field lives on a brane and gets its effective mass only from a field living on the other brane, the relevant interactions are suppressed by $M_{\rm P}$ leading [3] to the usual estimate $m_{\rm eff}^2 \sim H^2$.

JHEP05(2004)004

Our main purpose here is to consider, in more detail than before, the circumstances under which the mass of order H will in fact be generated by Planck-suppressed interactions, working as usual in the context of supergravity. We first consider the era of inflation, confirming that the effective mass will generically be of order H. We consider carefully the ways in which the mass of order H might be avoided for one or more fields. Then we go on to consider the situation after inflation, when the Universe is gaseous. In the case of matter domination, we confirm the effective mass of order H, including the case that the matter has spin greater than zero. Then we go on to consider the case of radiation domination, which has not previously been considered, and find that the generic mass is much less than H. (In all of these cases, the 'generic' value of the mass is coming from derivatives of the Kahler potential which are indeed generically present. We do point out that less generic terms in the superpotential may generate a bigger value mass.) We conclude by assessing the implication of our results.

2. Preliminaries

2.1 Defining the effective mass-squared of a real scalar field

If an effective field theory contains N real fields ϕ_i, the lagrangian of the fields is

$$\mathcal{L} = \sum G_{ij} \partial_\mu \phi_i \partial^\mu \phi_j - V(\phi_1, \phi_2, \ldots) \,. \tag{2.1}$$

The fields are coordinates in an N-dimensional field space with metric G_{ij}. As a result, one can in a sufficiently small region of field space, choose the fields so that $G_{ij} = \delta_{ij}$. (In our case 'sufficiently small' means 'much smaller than $M_{\rm P}$', since we are assuming that relevant non-renormalizable terms in the lagrangian are Planck-suppressed.) The fields then become cartesian coordinates in a flat space, and are defined when both the origin and the orientation of the axes is defined.

Usually the origin is chosen to be the vacuum expectation value (VEV) of the fields, corresponding to a minimum of V. Then the mass-squared matrix of the fields is defined as $m_{ij}^2 = \partial^2 V / \partial\phi_i \partial\phi_j$, and it may be convenient to choose the fields so that $m_{ij}^2 = \delta_{ij} m_i^2$ which then defines the masses-squared m_i^2.

If all or some of the fields in the effective theory are charged under a symmetry group with a fixed point, then it may be appropriate to choose that point as the origin. The masses-squared are defined in the same way as before, but they can now be either positive or negative (tachyonic). If any field has a tachyonic mass-squared, its VEV is not at the fixed point and some symmetry is spontaneously broken. The masses-squared defined with respect to the fixed point are then different from those defined with respect to the vacuum. In the Standard Model and the Minimal Supersymmetric Standard Model (MSSM), both definitions are used and one has to tell from the context which one is meant. For instance, the origin is taken to be the fixed point when one considers the running of masses-squared, from a high scale where they are perhaps all positive to a low scale where the Higgs masses-squared become tachyonic. If only a subset of the fields is charged under a symmetry group with a fixed point, all of this applies to that subset.

In some situations, one defines a field more generally as simply the distance along a chosen line in field space. To the extent that motion in the perpendicular directions can be ignored, a field defined in this way still has the dynamics of a canonically-normalized field. A familiar case is where a complex field $\Phi \equiv (\phi_1 + i\phi_2)/\sqrt{2}$ is written in the form $\Phi = r\exp(i\theta)$, and r has a nonzero VEV v. Then one considers the angular field $\phi = \sqrt{2}v\theta$.

This description of the possible ways of defining a field remains valid in the early Universe, if 'VEV' is replaced by 'a time-independent *or slowly-varying* value'. (The latter possibility arises because of Hubble-damping.) Among the additional possibilities, we mention the definition of the inflaton field as the distance along the slow-roll trajectory.

A useful definition of effective mass-squared for a field ϕ, at a given epoch in the early Universe, is simply

$$m_{\rm eff}^2 = V'' \,, \tag{2.2}$$

where a prime denotes a derivative with respect to ϕ, evaluated using the field values corresponding to the given epoch. (If a field is rapidly oscillating at the epoch, one should use its mean value.) It often happens that the potential is usefully expanded about a maximum,

$$V = V_0 - \frac{1}{2}\tilde{m}_{\rm eff}^2 \phi^2 + \cdots \,, \tag{2.3}$$

with the remaining terms small until a minimum at $\phi_{\rm min}$, and ϕ at the epoch in question being between the maximum and the minimum. Then the effective mass-squared $m_{\rm eff}^2 \equiv V''$ will typically be of order $\tilde{m}_{\rm eff}^2$ in magnitude.

The origin in eq. (2.3) need not be a fixed point of the symmetries. Indeed it cannot be if ϕ is a pseudo Nambu-Goldstone boson (PNGB), here defined as one for which some symmetry takes the form of a shift symmetry $\phi \to \phi + \text{const}$. Even if the origin is a fixed point, ϕ may not be one of the fields which diagonalise the mass matrix at the origin; that situation arises for the flat directions of the MSSM.

2.2 Matter fields and moduli

A given epoch in the history of the Universe corresponds to some point in field space, and at that epoch one can divide the fields into two types according to whether or not they are charged under symmetries having a fixed point which is nearby in Planck units. Adopting that point as the origin, canonically-normalized fields of the first type have values $\ll M_{\rm P}$ at the given epoch, while fields of the second type have values $\gtrsim M_{\rm P}$. Borrowing loosely from string terminology we may call them respectively matter fields and moduli. (The borrowing is loose, because the moduli space may have points of enhanced symmetry as will be discussed in a moment.)

For the matter fields, it is useful to choose the fixed point as the origin. Then the values of the canonically-normalized matter fields at the given epoch are, by definition, all much less than $M_{\rm P}$. As a result, it is enough to keep only low-order terms in the expansion of the potential in powers of the fields. The non-renormalizable terms of this expansion (dimension bigger than 4) will be suppressed by some mass scale, and we are interested in terms suppressed by $M_{\rm P}$.

JHEP05(2004)004

The moduli, by definition, do not transform under symmetries with a nearby fixed point. Examples suggested by the string theory, corresponding to canonically-normalized real fields, are as follows. First, the dilaton which may not transform under any symmetry at all. Next, fields corresponding to the size and shape of extra dimensions, or the distance between branes, which during the given epoch are at a distance of order $M_{\rm P}$ from a fixed point of their symmetry group.[2] Finally, string axion fields which possess a Z_2 shift symmetry, $\phi \to \phi + \Delta$ with $\Delta \sim M_{\rm P}$; in other words, the potential of these fields is periodic with period of order $M_{\rm P}$. (A given string axion field may be paired with a field of the first two types, to form a complex field in a chiral supermultiplet.) As far as we are aware, this list exhausts the possibilities suggested by string theory at the moment. Other possibilities might of course come up in the future, including ones which generate the widely-consider 'chaotic inflation' paradigm corresponding to an inflaton potential $m^2\phi^2$ out to $\phi \sim 10 M_{\rm P}$.[3]

The contributions of the moduli to the potential depend on the version of string theory adopted and even within a given version firm predictions are hard to come by [20]. However, a rather common expectation [21] is that the effective potential in the direction of a modulus ϕ is of the form

$$V(\phi) = \Lambda^4 \times f\left(\frac{\phi}{M_{\rm P}}\right), \tag{2.4}$$

with f and its low derivatives of order 1 in magnitude at a generic point within the expected range $0 \lesssim \phi \lesssim M_{\rm P}$ of the modulus.

Another common expectation, at least in the context of the heterotic string, is that the potential vanishes in the limit of unbroken SUSY. In the vacuum Λ may then be expected to be of order the SUSY-breaking scale $M_{\rm S}$, making the modulus mass-squared of order the gravitino mass-squared,

$$|m_{\rm eff}^2|^2 \sim m_{\frac{3}{2}}^2 = \frac{3M_{\rm S}^4}{M_{\rm P}^2}\,. \tag{2.5}$$

In the early Universe in the regime $\rho \gtrsim M_{\rm S}^4$, the SUSY breaking scale becomes ρ, which naively suggests $\Lambda \sim \rho^{\frac{1}{4}}$ corresponding to $|m_{\rm eff}^2| \sim H^2$. However, this conclusion is not inevitable, and in particular we shall show that during a radiation-dominated era $m_{\rm eff}^2$ can be much smaller than H^2. Going the other way, we note also that in other string theory schemes the moduli are supposed to be very heavy, so that even during inflation $|m_{\rm eff}^2| \gg H^2$.

[2]The symmetry group of the bulk moduli is supposed to be at least the group of modular transformations (a discrete gauge symmetry), with possibly additional symmetries corresponding to gauge and Yukawa interactions with either the Standard Model sector or a hidden sector. If such additional symmetries are present, the fixed point is said to be a point of enhanced symmetry. If a bulk modulus is at, or close to, a fixed point it is classified as a matter fields for the present purpose. The classification might change with time, so that for example inflation takes place near a fixed point which is at a Planckian distance from the vacuum.

[3]Such a potential has been derived [18] from Wilson line gauge symmetry breaking in a large extra dimension, but from the four-dimensional viewpoint this potential is generated by the one-loop (Coleman-Weinberg) potentials due to an infinite number of fields with arbitrarily high mass (a Kaluza-Klein tower). Hence it does not correspond to an effective four-dimensional field theory of the usual kind, which by definition contains only a finite number of fields with masses below the Planck scale. Also, it has been suggested [19] that this scenario has no known string-theoretic realization.

2.3 The supergravity potential

The lagrangian of $N = 1$ supergravity is determined by three functions (plus one or more Fayet-Iliopoulos constants). These are the Kahler potential K, superpotential W, and the gauge kinetic function. All three are functions of the complex scalar fields Φ_i, each of which corresponds to the scalar part of a chiral supermultiplet.

Possible set of the Planck-suppressed operators depends on the model since some the operators may be forbidden or suppressed by some symmetry [22]. The superpotential and the gauge kinetic function are holomorphic functions of the chiral superfields Φ_i, and hence it is relatively easy to find a symmetry which forbids some class of operators. (Of course, it is unclear if such a symmetry is respected by the physics at the Planck scale; if not, Planck-suppressed operators show up.) On the contrary, it is more difficult to control the Kahler potential since it depends on the Φ_i and their complex conjugates $\bar{\Phi}_i$.

The supergravity potential is of the form

$$V = V_F + V_D\,. \tag{2.6}$$

We shall be mostly interested in the F-term, which is given by

$$V_F = e^{K/M_{\rm P}^2}\left(D_i W K^{i\bar{j}} \overline{D_j W} - \frac{3|W|^2}{M_{\rm P}^2} \right). \tag{2.7}$$

Subscripts for Kahler and superpotentials denote partial derivatives with respect to corresponding fields, $K^{i\bar{j}}$ is the inverse of the Kahler metric $K_{i\bar{j}}$, and

$$D_i W \equiv W_i + \frac{W K_i}{M_{\rm P}^2}\,. \tag{2.8}$$

It is useful to separate eq. (2.7) into two parts,

$$V_F = \tilde{M}_{\rm S}^2 - \frac{3 e^{K/M_{\rm P}^2}|W|^2}{M_{\rm P}^2}\,. \tag{2.9}$$

The first term spontaneously breaks supersymmetry, and we called it $\tilde{M}_{\rm S}^4$ because its vacuum value is usually denoted by $M_{\rm S}^4$. In the vacuum the two terms accurately cancel for some mysterious reason (the cosmological constant problem).

3. During inflation

3.1 The effective mass-squared during inflation

We focus on the usual case that the inflaton potential is dominated by the F-term, except for a few remarks on D-term inflation at the end of the section. In that case, it has long been known [4] that Planck-suppressed interactions generically give $m_{\rm eff}^2 \sim \pm H_*^2$, for each scalar field whose true mass is less than the inflationary Hubble parameter H_*.[4] We here consider the situation more carefully than before, to better understand the circumstances under which $|m_{\rm eff}^2|$ might instead be much bigger or much smaller than H_*^2.

[4]Such fields are expected to exist, since according to existing inflation models H_* is at least of order the gravitino mass $m_{3/2}$ [23], while one may expect that at least some scalar fields get their mass from gravity-mediated SUSY breaking corresponding to $m_\phi \sim m_{3/2}$.

A simple argument due to Stewart [24] shows that $m^2_{\rm eff}$ defined by eq. (2.2) is generically *at least* of order H^2. Focusing on a particular time, we can choose the origin in field space so that all fields vanish at that time. Demanding that each field is canonically normalized (part of the definition of $m^2_{\rm eff}$), the power series expansion of the Kahler potential in Φ_i and $\bar\Phi_i$ must then contain the terms

$$K \supset \sum_i \pm|\Phi_i|^2 \,, \tag{3.1}$$

leading indeed to canonical normalization at the origin,

$$\sum K_{i\bar j}\partial^\mu\Phi_i\partial_\mu\bar\Phi_j = \sum \partial^\mu\Phi_i\partial_\mu\bar\Phi_i \,. \tag{3.2}$$

Through the factor $\exp(K/M_{\rm P}^2)$ in eq. (2.7), the term eq. (3.1) gives a contribution $\pm V/M_{\rm P}^2 = \pm 3H^2$ to the effective masses-squared of each field. In the generic case this contribution will not be canceled by other contributions, leading as advertised to $|m^2_{\rm eff}|$ at least of order H^2.

We would like to point out that the sign in eq. (3.1) need not be positive as is usually assumed. The sign is a property of the effective supergravity theory, to be determined presumably by string theory which seems to have no prejudice towards the positive sign. In particular, a commonly occurring approximation (for instance in the untwisted sector for the weakly coupled heterotic string) is $K \simeq M_{\rm P}^2\ln(1-\sum_i|\Phi_i|^2/M_{\rm P}^2)$, which gives the negative sign.

The above argument is important because it does not invoke an interaction term in either K or W, relying instead on the form of the supergravity potential to generate a particular Planck-suppressed term in the potential V. Additional Planck-suppressed terms in V will in general be generated by Planck-suppressed terms in K and/or W, some of which may contribute to the effective mass-squared. Keeping the convention that the origin of field space corresponds to the values of the fields at the time under consideration, there may be terms contributing to K the amount

$$\Delta K = \lambda_i\Phi_i^2 + {\rm c.c.} \tag{3.3}$$

Taking for definiteness λ_i either real or imaginary, these terms contribute to the mass-squared of respectively the real and imaginary parts of Φ_i and amount $\pm|\lambda_i|H^2$. To understand the significance of such terms, it is helpful to transfer them to the superpotential using the Kahler transformation which leaves invariant the form of the supergravity potential;

$$K \to K - \Delta K \,, \tag{3.4}$$

$$W \to e^{\lambda_i\Phi_i^2/M_{\rm P}^2}W \,. \tag{3.5}$$

We see that the terms we are considering are equivalent to Planck-suppressed interaction terms in the superpotential, which means that we expect $|\lambda_i| \lesssim 1$ as far as the cutoff scale of the higher dimensional operators is the Planck scale. Therefore, these terms are expected to contribute at most of order $\pm H^2$ to the effective masses-squared. Notice that the interaction given in eq. (3.3) can be forbidden by some symmetry if Φ_i has some non-trivial transformation property. If the Φ_i^2 term in the Kahler potential is consistent with all the symmetry, on the contrary, mass term of Φ_i may be also allowed in the superpotential (unless R-symmetry is imposed). Importantly, natural scale of the mass of Φ_i in this case is of order the cutoff scale (i.e., $M_{\rm P}$ in our case) if there is no suppression for the interaction of Φ_i. If so, the (effective) mass of ϕ is always dominated by the superpotential contribution and the contribution of the Kahler interaction is sub-dominant.

Now we discuss more generally the effect of Plank-suppressed interaction terms, considering only the effective masses-squared of the matter fields so as to take on board the possible effect of symmetries. Our analysis goes in some respects a little beyond the one of [6].

To take into account the possible effect of the moduli on the masses-squared of the matter fields, we shall assume a potential of the form eq. (2.4) for the moduli. Since the matter field values are small on the Planck scale, one need keep only low-order terms in the power series expansions of K and W. Each term in W is holomorphic, and it is useful to adopt the convention [25] that K has no holomorphic terms (possible since such terms can always be transferred to W by a Kahler transformation). The terms considered in eq. (3.1) will then be the leading ones,

$$K = \sum_i \pm|\Phi_i|^2 + \cdots . \tag{3.6}$$

Through the term $\exp(K/M_{\rm P}^2)$, these terms will generate for each field a contribution $\pm 3H^2$ to $\tilde{m}_{\rm eff}^2$, as we already discussed.

To this contribution effect of the non-renormalizable terms in K and W has to be added. To discuss them, we suppose for simplicity that a single term $i = I$ dominates the supergravity potential eq. (2.7), and we also make the reasonable assumption $|K| \lesssim M_{\rm P}^2$. The generic estimate $\tilde{m}_{\rm eff}^2 \sim \pm H^2$ comes from the contribution of non-renormalizable terms in K. It is analogous to the contribution which, in the vacuum, generates soft masses-squared of order $\pm M_{\rm S}^4/M_{\rm P}^2$. During inflation, the corresponding contribution is $\pm \tilde{M}_{\rm S}^4/M_{\rm P}^2$, which indeed is expected to be of order $\pm H^2$ because the terms of eq. (2.9) are not expected to cancel as they do in the vacuum.

The field ϕ whose mass-squared we are estimating will generally be either the radial or the angular component of some complex field Φ. Except for some remarks at the end, we take it to be the radial component. Assume first that Φ is charged only under U(1) symmetries, which act on its phase. None of these symmetries can prevent a contribution

$$K^{I\bar{I}} \supset \frac{\lambda|\Phi|^2}{M_{\rm P}^2} . \tag{3.7}$$

The coupling $|\lambda|$ is generically expected to be of order 1, giving indeed a contribution of order $\pm H^2$ to $\tilde{m}_{\rm eff}^2$. If Φ_I is a matter field, the contribution eq. (3.7) comes from a quartic term in the power series expansion, [26, 27, 6]

$$K \supset \frac{\lambda|\Phi_I|^2|\Phi|^2}{M_{\rm P}^2} . \tag{3.8}$$

If instead Φ_I is a modulus, the same estimate follows if we invoke eq. (2.4) so that

$$K \supset f\left(\frac{\phi_I}{M_{\rm P}}\right)|\Phi|^2\,, \tag{3.9}$$

where ϕ_I is the real or imaginary part of Φ_I.

If instead Φ is charged under continuous non-abelian symmetries, one can replace $|\Phi|^2$ by an invariant sum $\sum_i |\Phi_i|^2$ or one can replace Φ by an invariant product of fields. Then the term eq. (3.7) is, again, not forbidden by any symmetry. On the other hand, if the non-abelian symmetry is global it still makes sense to consider Φ itself, which becomes a PNGB if the global symmetry is spontaneously broken [28].

If we make the reasonable assumption that the second term of eq. (2.9) is actually negligible, with also $|K_I| \lesssim M_{\rm P}$, then the contribution eq. (3.7) to $\tilde{m}^2_{\rm eff}$ is actually the only one coming from Planck-suppressed terms in K. Aside from this contribution, the effective masses-squared may receive contributions of order $\pm H^2$ from the terms we discussed after eq. (3.3), but since this can be regarded as a contribution to W it may be forbidden by symmetries for the matter fields that we are now considering.

Typically, there are no other contributions to the effective masses-squared which are of order $\pm H^2$. An exception arises if the superpotential is proportional to the field I. Then, $\tilde{m}^2_{\rm eff}$ for the field I receives additional contributions of order $\pm H^2$. If (i) the plus sign of eq. (3.6) holds, and (ii) $|K_i| \ll M_{\rm P}$ for every field [10], these additional contributions cancel [5] those from the factor $\exp(K/M_{\rm P}^2)$, leaving only the contribution from eq. (3.7). (The first condition has not been noted before.) Then one can achieve $|\tilde{m}^2_{\rm eff}| \ll H^2$ by supposing that $|\lambda|$ in eq. (3.7) is suppressed. This idea has been widely adopted as a paradigm for inflation [10], but it is incompatible with the existence of moduli having a Kahler potential of the form eq. (2.4), since that will give $|K_i| \sim M_{\rm P}$ leading to additional contributions [10] to $\tilde{m}^2_{\rm eff}$.

Aside from the above exception, interaction terms in W give a contribution to $\tilde{m}^2_{\rm eff}$ that has nothing to do with H^2. This contribution involves only fields with non-zero values, and so may be regarded as a generalization of the Higgs effect (which, in the vacuum, generates the true mass of a particle through its gauge coupling to a nonzero field). We discussed already (eq. (1.5)) the effect of a renormalizable term, and similar considerations apply to non-renormalizable terms. Consider

$$W \supset \frac{\lambda}{2M_{\rm P}}\Psi^2\Phi^2\,, \tag{3.10}$$

with $|\lambda| \sim 1$, and Ψ any field with a nonzero value. It contributes to the potential terms

$$V \supset \frac{\lambda|\Psi|^4|\Phi|^2}{M_{\rm P}^2} + \frac{\lambda|\Phi|^4|\Psi|^2}{M_{\rm P}^2}\,. \tag{3.11}$$

This will give a contribution $m^2_{\rm eff} = \lambda|\Psi|^4/M_{\rm P}^2$, which could be bigger than H^2. However, similarly to the situation we discussed after eq. (1.5), this contribution will not appear if $|\Phi|$ is large because then Ψ will have a large positive mass-squared which drives it to zero. In addition, notice that it may also be possible to forbid or suppress the term eq. (3.10) using symmetry arguments.

All of this concerns the case that ϕ is the radial component of Φ. We end this section by briefly considering the opposite situation, that ϕ corresponds to the angular component of Φ with the radial component fixed or slowly-varying [6, 29]. The term $\exp(K/M_{\rm P}^2)$ does not contribute to the $m_{\rm eff}^2$ of the angular field, but non-renormalizable terms in K generate $m_{\rm eff}^2$ through a generalized A-term. In the absence of symmetries one again finds $m_{\rm eff}^2 \sim \pm H^2$, but now the mass can easily be suppressed by invoking symmetries.

3.2 Suppressing the mass-squared during inflation

As we already noted, at least the field responsible for the primordial curvature perturbation must have $|m^2| \lesssim 0.1H^2$, somewhat smaller than the generic expectation. This could of course be an accident, but there has been a lot of discussion about how instead the suppression might be very strong, and come about for a definite reason. Depending on the mechanism, the strong suppression might hold only for very special fields, for a wide class of fields or for all fields. We here briefly recall that discussion, emphasizing some points which were perhaps previously obscure and some possible new directions for research.

Considering some particular field ϕ with effective mass $m_{\rm eff}$, suppose first that it is a pseudo-Nambu-Goldstone-boson (PNGB). This means that the lagrangian is approximately invariant under a global symmetry which includes the shift symmetry $\phi \to \phi + \text{constant}$. The mass $m_{\rm eff}$ vanishes in the limit of unbroken shift symmetry, and with sufficiently weak symmetry breaking it will in general be less than H during inflation.[5]

If ϕ is the only field affected by the shift symmetry, it can be realized by choosing K to be a function only of $\Phi + \bar{\Phi}$ and W to be either independent of Φ or else of the form $\exp(\text{const} \times \Phi)$. The axionic components of string moduli are supposed to be examples of this. Little seems to be known about the manner in which the shift symmetry is broken in this case [38], but one may hope that it will be sufficiently mild to avoid the mass of order H for the string axions. It should be noted that the flatness of the potential of a PNGB may be protected by the symmetry over the entire range of field values, not just near the origin at which the mass-squared is defined. In other words, the coefficients of higher powers of the field may also be controlled. Whether this happens or not depends of course on the way in which the symmetry is broken.

Instead of imposing a symmetry on the whole lagrangian, one can suppress the mass of one or more scalar fields by choosing special forms for K and W which do not correspond to a symmetry, but which *during inflation* keep the potential perfectly flat in the limit that these forms hold exactly. In other words, one can demand that the fields under consideration are effectively PNGB's during inflation. A general recipe for doing this was given by Stewart [24], the first step of which is to invoke an R-parity which keeps $W = 0$ during inflation so that eq. (2.7) takes the simple form

$$V = e^{K/M_{\rm P}^2} K^{i\bar{j}} W_i W_j^* \,. \tag{3.12}$$

[5] An important exception can arise in the case of the inflaton in a non-hybrid model ('Natural Inflation' [30]). Then the inflationary potential vanishes in the limit of unbroken symmetry, and so does H. As a result the symmetry does not generally help to keep m/H small in this case [31]. Even then though, one can have $H/m \to 0$ in an appropriate limit of unbroken symmetry if symmetry breaking is controlled by two independent parameters. This is what happens in the case of gauge inflation [18].

A specific realization of this recipe in the context of the weakly coupled heterotic string was given in [39]. This realization works with the untwisted sector fields Φ_{Ii} and the modulus T_I (with index I being specifying the modulus), and assumes that in a sufficiently good approximation the Kahler potential depends only on the quantities

$$x_I \equiv T_I + \bar{T}_I - \sum_i \frac{|\Phi_{Ii}|^2}{M_{\rm P}^2} \,, \tag{3.13}$$

which determine the radii of compactification of the three tori. By virtue of this assumption, the Kahler potential possesses the 'Heisenberg invariance' $\Phi_{Ii} \to \Phi_{Ii} + \epsilon_{Ii}$, $T_I \to T_I + \sum_i \epsilon^*_{Ii} \Phi_{Ii}$, which would be shift symmetries for the untwisted fields if the superpotential did not depend on them.

The PNGB and 'effective PNGB' possibilities have been explored both for the inflaton [31, 28, 18, 40, 41, 32, 33, 36, 37] (see also [34] for the indirect use of a PNGB) and for the the curvaton [12, 35, 29], and they constitute at present a very active area of research.

Prior to [39] a different and simpler scheme was proposed [42, 43], working with an overall modulus T and K a function only of $x \equiv (T+\bar{T}-\sum |\Phi_i|^2/M_{\rm P}^2)$ the sum running over the untwisted fields. Unfortunately there does not seem to be any well-motivated form for K which allows this scheme to work. The original idea was to use the 'no-scale' paradigm, according to which $K = -3M_{\rm P}^2 \ln x$, and W depends only on the untwisted fields. This leads to

$$V = \frac{1}{3x^2} \sum |W_i|^2 \,. \tag{3.14}$$

In the vacuum, $V = 0$, and no soft masses are generated because V is the same as for unbroken global supersymmetry except for the first factor. (The no-scale case is not generally thought to be a good approximation to reality, but we do not enter into that issue here.) But during inflation, where V is non-vanishing, the no-scale form cannot work because x will run away driving V to zero. The situation could be rescued if K had an additional term depending only on x but existing proposals are ad hoc [42], except for that of [43] which invokes a quite special loop correction. The latter requires the last term of eq. (2.7) to accurately cancel V_F, and no inflation model has been exhibited in which it works.

Finally, the mass of a field may be suppressed by the quantum correction, provided that the field has unsuppressed interactions, and global supersymmetry is a good approximation with $m_{\rm eff}$ a soft mass. In that case, $m_{\rm eff}^2$ becomes ϕ-dependent (a running mass) and may pass through zero. In the vicinity of the zero, the potential will have a maximum or minimum with $m_{\rm eff} \ll H$. This possibility has been explored so far only for the inflaton [44]–[47] but it may also make sense to consider it for the curvaton in the case that the potential develops a minimum. (After inflation the interactions of the curvaton should be suppressed so that it does not decay promptly, but that need not be the case during inflation.)

We end by mentioning the case of D-term inflation [24, 48, 49]. If the D-term dominates completely and the gauge kinetic function has negligible dependence on the inflaton field, the inflaton potential is given by the one-loop potential of the waterfall field (i.e., the inflaton) S and there is no mass term. One potential problem of this scenario is that, if the superpotential or the gauge kinetic function depends on S through Planck-suppressed

JHEP05(2004)004

terms, Planck-suppressed interactions significantly affect the inflaton potential since the inflaton field value is of order M_{P} when cosmological scales leave the horizon [2, 50]. One possibility of eliminating such Planck-suppressed interactions is to introduce some symmetry under which S transforms. Such (global) symmetry may be, however, violated once the effects of the quantum gravity is taken into account. Also, with a negligible F term it is difficult to see how the dilaton can be stabilized [45]. So it is unclear if a realistic model of the D-term inflation can be really constructed.

Let us summarize this section. During inflation, the mass of order H for a given field can be avoided if the field is a PNGB, or if its mass runs, or if the potential during inflation has a special string-inspired form. It is also avoided if inflation occurs with the F term completely negligible (D-term inflation) though, in such a case, the model should be arranged such that all the dangerous Planck-suppressed interactions somehow vanish. In addition, we have also seen that, contrary to what is often stated, no-scale supergravity does not avoid the mass of order H.

4. After inflation

Now we consider what happens after inflation. According to present ideas, the Universe after inflation is supposed to be gaseous save for exceptional epochs. Possibilities for the latter include brief phase transitions (electroweak for instance) and of course brief further inflation (such as thermal inflation). These, though, typically occur after H falls below m in which case they are irrelevant in the present context.

More relevant is the epoch immediately after inflation, when the field(s) responsible for the inflationary potential oscillate. In a non-hybrid model there is just the inflaton field. In that case, if the field does not decay rapidly, the oscillation after at most a few Hubble times will usually become almost sinusoidal. Then it is equivalent to a matter-dominated gas on timescales much bigger than the inverse effective mass of the inflaton. Instead though, the inflaton oscillation may efficiently create particles through non-perturbative effects generally known as preheating. Also, for hybrid inflation the waterfall field oscillates and interacts with at least the inflaton field. The situation immediately after inflation may therefore be very complicated. However, one still expects to find again a gas after a Hubble time or so, consisting of particles created by the oscillation plus maybe a single oscillating field. The one exception might be the case where the interacting inflaton and the waterfall fields oscillate with negligible decay; we ignore this case for the moment.

We thus proceed on the assumption that the Universe after inflation consists of a gas and/or one or more non-interacting and oscillating homogeneous scalar fields, the latter being equivalent to a matter-dominated gas. This means in practice that the total energy density is either radiation- or matter-dominated, since transitions from one case to the other will always be rather brief.

The particle species making up the gas may have unsuppressed interaction terms, but the particles in the gas are supposed to be moving freely which means that the expectation values of such interaction terms are negligible. This means that among the terms in the lagrangian which involve the gas particles, only the kinetic terms $\mathcal{L}_{\rm kin}$ and the mass terms $\mathcal{L}_{\rm mass}$ can have significant expectation values.

We are interested in the effective mass-squared of a scalar field ϕ, which does not correspond to any of the gas particles, and which has only Planck-suppressed interactions with those particles. We are considering an epoch when H is bigger than the true mass m_ϕ, and we ask whether Planck-suppressed interactions induced by K will lead to an effective mass-squared of order $\pm H^2$.

We begin by recalling some results from the theory of free fields, which follow from the fact that the lagrangian is essentially that of uncoupled harmonic oscillators. First, the kinetic and mass terms of the lagrangian are equal by virtue of the field equations;

$$\langle \mathcal{L}_{\rm kin} \rangle = - \langle \mathcal{L}_{\rm mass} \rangle \,. \tag{4.1}$$

Next, the energy density of the gas is of the form

$$\rho = \rho_{\rm kin} - \langle \mathcal{L}_{\rm mass} \rangle \,, \tag{4.2}$$

the first term involving spacetime derivatives. For radiation domination the mass term is negligible so that

$$|\langle \mathcal{L}_{\rm mass} \rangle| \ll \rho = 3 H^2 M_{\rm P}^2 \,. \tag{4.3}$$

For matter domination, spatial derivatives give a negligible contribution to $\rho_{\rm kin}$, and as a result $\rho_{\rm kin} = \langle \mathcal{L}_{\rm kin} \rangle$, leading to

$$|\langle \mathcal{L}_{\rm mass} \rangle| = \frac{1}{2} \rho = \frac{3}{2} H^2 M_{\rm P}^2 \,. \tag{4.4}$$

Next, we use the well-known expressions for the mass and kinetic terms of the supergravity lagrangian, to estimate the magnitude of those interaction terms in the lagrangian which are proportional to ϕ^2, and are also proportional to either $\mathcal{L}_{\rm kin}$ or $\mathcal{L}_{\rm mass}$ so that they can have a significant expectation value.

Consider first terms proportional to $\mathcal{L}_{\rm mass}$. If the mass term is that of a scalar field, it comes from the potential eq. (2.7) so that the factor $e^{K/M_{\rm P}^2}$ generically gives an interaction term

$$\mathcal{L}_{\rm massint} \sim \pm \frac{\phi^2}{M_{\rm P}^2} \mathcal{L}_{\rm mass} \,. \tag{4.5}$$

Considering instead chiral fermion, gaugino or the gravitino, all of their mass terms come with a prefactor $e^{K/2M_{\rm P}^2}$, giving the same interaction except for a factor $\frac{1}{2}$. In the case of scalar particles, we noted before that higher-dimensional terms in K could lead to contributions comparable with the one coming from the exponential factor, and an examination of the mass term for chiral fermions leads to the same conclusion. With such terms, eq. (4.5) still provides an estimate of the interaction terms proportional to ϕ^2. For massive gauge boson which acquires mass from the Higgs mechanism, interaction term like (4.5) may also exist. Mass term of the gauge boson originates from the kinetic term of the Higgs boson. Thus, if the chiral multiplet of the Higgs Φ_H has a coupling to Φ in the Kahler potential with the form $\sim |\Phi_H|^2 |\Phi|^2 / M_{\rm P}^2$, interaction term like (4.5) shows up from the Kahler metric after the Higgs boson acquires the VEV.

JHEP05(2004)004

Now consider terms proportional to $\mathcal{L}_{\rm kin}$. If the kinetic term is that of a scalar or of a chiral fermion, it arises because the prefactor of the kinetic term is $K_{i\bar{j}}$. This means that Planck-suppressed interaction terms of the form $K = |\Phi_{\rm gas}|^2|\Phi|^2/M_{\rm P}^2$ (with $\Phi_{\rm gas}$ being chiral multiplet of the particles in the gas) will generate an interaction term associated with the kinetic term

$$\mathcal{L}_{\rm kinint} \sim \pm\frac{\phi^2}{M_{\rm P}^2}\mathcal{L}_{\rm kin} \,. \tag{4.6}$$

If instead the kinetic term is that of a gaugino or a gauge boson, a similar conclusion follows if the gauge kinetic function contains a term like $\sim \Phi^2/M_{\rm P}^2$. Notice, however, that it is model-dependent if such a term exists since it may vanish because of some symmetry.

To summarize, we have found that the terms in the lagrangian relevant for determining the effective mass-squared of a field ϕ are

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + \left(1-\lambda_{\rm kin}\frac{\phi^2}{2M_{\rm P}^2}\right)\mathcal{L}_{\rm kin}(\chi) + \left(1-\lambda_{\rm mass}\frac{\phi^2}{2M_{\rm P}^2}\right)\mathcal{L}_{\rm mass}(\chi)\,, \tag{4.7}$$

with $\lambda_{\rm kin}$ and $\lambda_{\rm mass}$ expected to be of order ±1. Here χ stands for all of the fields corresponding to particle species in the thermal bath, and ϕ is the scalar field whose effective mass is to be evaluated.

With this preparation, the results are immediate. Using eqs. (4.1), (4.4) and (4.7) one finds during matter domination the advertised result $m_{\rm eff}^2 \sim \pm H^2$. Using instead eqs. (4.1), (4.3) and (4.7), one finds during radiation domination $|m_{\rm eff}^2| \ll H^2$. In fact, more precise study of $m_{\rm eff}^2$ is possible by calculating the expectation values of the relevant operators in radiation- or matter-dominated universe. For details, see appendix A.

As in the case of inflation, an additional contribution to the effective mass-squared could come from terms in the superpotential. Any such term would contribute to both the scalar and fermion fields of a chiral supermultiplet. Consider in particular the term in eq. (3.10). The discussion in that case follows the lines of the one we already gave for the case of inflation, except that we now need to consider also the fermionic partner of Ψ which we denote by χ. The terms contributing to the effective mass-squared of Φ is

$$\mathcal{L} \supset -\lambda\phi^2\left(\frac{|\Psi|^4}{M_{\rm P}^2} + \frac{\bar{\chi}\chi}{M_{\rm P}}\right)\,, \tag{4.8}$$

with $\lambda \sim \pm1$. If the gas consists of Ψ particles, the first term is the relevant one, and will contribute

$$|m_{\rm eff}^2| \supset M_{\rm P}^{-2}\langle|\Psi|^4\rangle\,, \tag{4.9}$$

which will be bigger than H^2 if $\langle|\Psi|^4\rangle \gtrsim \rho$. If instead the gas consists of χ particles, eq. (4.8) will contribute

$$|m_{\rm eff}^2| \supset M_{\rm P}^{-1}\langle\bar{\chi}\chi\rangle = \frac{(M_{\rm P}/m_\chi)\langle\mathcal{L}_{\rm mass}\rangle}{M_{\rm P}^2}\,, \tag{4.10}$$

which is definitely much bigger than H^2. However, as in the discussion after eqs. (1.5) and (3.11), these contributions to the effective mass-squared of ϕ may be absent if ϕ is large, because Ψ and χ will then acquire large masses. If large enough, such masses will prevent the creation of Ψ and χ particles, so that they will not be a component of the gas and will not contribute to $m_{\rm eff}^2$. Alternatively, the contribution may be suppressed or forbidden by a symmetry.

To summarize, we have considered generic Planck-suppressed interactions coming from the Kahler potential, and have found that these generate a mass-squared of order $\pm H^2$ during matter-domination but a much smaller one during radiation domination. Just as in the case of inflation, further contributions to the effective mass-squared may come from Planck-suppressed terms in W, involving large slowly-varying fields (a generalization of the Higgs effect). In addition, a similar effect might come from rapidly oscillating scalar fields which represent constituents of the gas. Consider, for example, the following superpotential

$$W = y \Psi_1 \Psi_2 \Psi_3 \,, \tag{4.11}$$

where Ψ_i are the chiral superfields for the (scalar) particles in the gas and y is the coupling constant. Then, taking account of the effects of the Kahler potential, the scalar potential may contain the term of the form

$$V = y^2 \left(1 + \lambda \frac{|\Phi|^2}{M_{\rm P}^2} \right) |\Psi_1 \Psi_2|^2 + \cdots \,. \tag{4.12}$$

From this potential, effective mass-squared of Φ receives a contribution

$$m_{\rm eff}^2 = \frac{\lambda y^2}{M_{\rm P}^2} \langle |\Psi_1|^2 \rangle \langle |\Psi_2|^2 \rangle + \cdots \,. \tag{4.13}$$

If y is small enough, Ψ_i (approximately) obeys the equation of motion of the massless particles. Then, in the thermal bath, we obtain $\langle |\Psi_i|^2 \rangle \sim T^2$ (see appendix A.) If $y \ll 1$ (or $\lambda \ll 1$), $m_{\rm eff}$ is smaller than H. If $y \sim 1$, on the contrary, effective mass may receive a contribution comparable to H. For the case of $y \sim 1$, however, one should note that we would no longer be dealing with a gas of free particles (ideal gas) since the interaction becomes very strong. Thus, in this case, the situation is different from the usual 'radiation-dominated' or 'matter-dominated' epoch.

Finally, we consider the possibility that the mass of order H during matter domination might be suppressed. As with inflation, this is certainly possible if the relevant field ϕ is a PNGB, and suppression may also occur accidentally. In contrast with inflation though, it is difficult to find other ways of suppressing the mass, because the scalar fields cannot be taken to be time-independent. As a result, special choices of K and W are unlikely to work, while a running mass will pass through zero only at a special epoch.

5. Conclusion

The considerations of this paper apply if there are scalar fields whose true mass is less than the inflationary Hubble parameter H_*. One expects that this will be the case if H_* is of order the gravitino mass (low-scale inflation) but not if it is much bigger. Under that assumption, we considered the effective mass of a scalar field ϕ, whose true mass m_ϕ is indeed less than H_*, and which has only Planck-suppressed interactions with the fields responsible for the energy density of the Universe.

We have considered the era of inflation, and also the subsequent era on the reasonable assumption that the Universe is then either matter- or radiation-dominated. For inflation and matter domination, we have confirmed the received wisdom that the effective mass-squared is generically of order $\pm H^2$. We have also noted that this result is quite difficult to avoid unless the relevant field is a PNGB, especially in the case of matter domination. We then considered the case of radiation domination, finding the perhaps unexpected result that the effective mass-squared will be much less than H^2 if the interaction among the particles consisting of the radiation are weak enough.

These results have an important implication for the curvaton paradigm, according to which the primordial density perturbation only at some time after inflation ends, through the action of some 'curvaton' field. For this paradigm to be viable, the perturbation of the curvaton field which is generated during inflation needs to be maintained until the curvature perturbation is generated, which probably requires [17] that the curvaton does *not* have a mass of order H between the end of inflation and the epoch when the curvature perturbation is generated. This can be achieved by making the curvaton a PNGB [35], but according to our result it can also be achieved by having inflation give way promptly to radiation domination. Of course such early reheating, combined with the high inflation scale that we are assuming, will lead to a copious production of gravitinos, but as is well know these can be sufficiently diluted by later entropy production coming from the decay of a long-lived particle possible preceded by thermal inflation.

Acknowledgments

T.M. would like to thank the organizers of COSMO03 workshop, where this work was initiated, for their hospitality. D.H.L. would like to thank Tom Banks, Michael Dine, Renata Kallosh and Andrei Linde and Mary K Gaillard for useful exchanges. T.M. is supported by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports, and Culture of Japan, No. 15540247.

A. A more precise analysis

In this appendix, we give a more precise analysis of the effective mass-squared generated by a gas, making contact with the result of finite-temperature effective field theory. Here, we consider the lagrangian of the form eq. (4.7), where χ is the particle in the thermal bath while ϕ is the scalar field whose effective mass is to be evaluated. Notice that, with the Planck-suppressed interactions in the Kahler potential discussed in the previous section, $|\lambda_{\rm mass}| \sim 1$ and, if χ is in a chiral multiplet, $|\lambda_{\rm kin}| \sim 1$.

Consider first the case of a scalar field χ; then, $\mathcal{L}_{\rm kin}$ and $\mathcal{L}_{\rm mass}$ are given by

$$\mathcal{L}_{\rm kin}(\chi) = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi\,, \tag{A.1}$$

$$\mathcal{L}_{\rm mass}(\chi) = -\frac{1}{2}m_\chi^2\chi^2\,. \tag{A.2}$$

We expand the field operator χ using the creation and annihilation operators $a_{\mathbf{p}}$ and $a_{\mathbf{p}}^\dagger$. Here, we use the box normalization of the wave functions using the box with the volume $V = L^3$. Then, we obtain

$$\chi(x) = \sum_{\mathbf{p}} \frac{1}{\sqrt{2E_{\mathbf{p}} V}} \left(a_{\mathbf{p}} e^{-ipx} + a_{\mathbf{p}}^\dagger e^{ipx} \right) , \tag{A.3}$$

where the four momentum $p = (E_{\mathbf{p}}, \mathbf{p})$ obeys the on-shell condition $k^2 = m_\chi^2$. (Here, we expect that the time scale to realize the equilibrium is much shorter than the cosmic time scale $\sim H^{-1}$, and hence we neglect the effect of red-shift in evaluating the expectation values.) In addition, spatial components of the momentum is given in the form $\mathbf{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$ with n_x, n_y, and n_z being integers. Notice that, with this normalization, $[a_{\mathbf{p}}, a_{\mathbf{p}'}^\dagger] = \delta_{\mathbf{p}\mathbf{p}'}$.

Substituting eq. (A.3) into eqs. (A.1) and (A.2), expectation values of $\mathcal{L}_{\rm kin}$ and $\mathcal{L}_{\rm mass}$ are given by

$$\langle \mathcal{L}_{\rm kin} \rangle = \frac{1}{V} \sum_{\mathbf{p}} \frac{k^2}{2E_{\mathbf{p}}} \langle a_{\mathbf{p}}^\dagger a_{\mathbf{p}} \rangle , \qquad \langle \mathcal{L}_{\rm mass} \rangle = -\frac{1}{V} \sum_{\mathbf{p}} \frac{m_\chi^2}{2E_{\mathbf{p}}} \langle a_{\mathbf{p}}^\dagger a_{\mathbf{p}} \rangle . \tag{A.4}$$

With the replacement $\frac{1}{V}\sum_{\mathbf{p}} \to \int \frac{d^3\mathbf{p}}{(2\pi)^3}$ and $k^2 = m_\chi^2$, the above formulae become

$$\langle \mathcal{L}_{\rm kin} \rangle = -\langle \mathcal{L}_{\rm mass} \rangle = \frac{1}{2} m_\chi^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\langle n_\chi(\mathbf{p}) \rangle}{E_{\mathbf{p}}} , \tag{A.5}$$

where $\langle n_\chi(\mathbf{p}) \rangle = \langle a_{\mathbf{p}}^\dagger a_{\mathbf{p}} \rangle$ is the expectation value of the number density of χ with the momentum $\mathbf{p}$. Using the relation $\langle \mathcal{L}_{\rm kin} \rangle = -\langle \mathcal{L}_{\rm mass} \rangle$, we obtain

$$m_{\rm eff}^2 = \frac{1}{M_{\rm P}^2} (\lambda_{\rm kin} - \lambda_{\rm mass}) \langle \mathcal{L}_{\rm kin} \rangle . \tag{A.6}$$

Also, the energy density is

$$\rho = \int \frac{d^3\mathbf{p}}{(2\pi)^3} E_{\mathbf{p}} \langle n_\chi(\mathbf{p}) \rangle . \tag{A.7}$$

During matter domination, the random motion of the gas of χ particles is non-relativistic, which means that $\langle n_\chi(\mathbf{p}) \rangle$ is suppressed unless $|\mathbf{p}| \ll m_\chi$, and

$$\langle \mathcal{L}_{\rm kin} \rangle \simeq \frac{1}{2} \rho_\chi \simeq \frac{1}{2} m_\chi \int \frac{d^3\mathbf{p}}{(2\pi)^3} \langle n_\chi(\mathbf{p}) \rangle . \tag{A.8}$$

Denoting the energy fraction of χ as Ω_χ, Δm_ϕ^2 is given by

$$m_{\rm eff}^2 = (\lambda_{\rm kin} - \lambda_{\rm mass}) \frac{\rho_\chi}{2M_{\rm P}^2} = \frac{3}{2} (\lambda_{\rm kin} - \lambda_{\rm mass}) \Omega_\chi H^2 . \tag{A.9}$$

Thus, during a matter-dominated era when the non-relativistic gas of χ dominates the Universe, an effective mass of $O(\pm H^2)$ is generated, provided that $\lambda_{\rm kin}$ and $\lambda_{\rm mass}$ are of $O(1)$.

JHEP05(2004)004

During radiation domination, the random motion of the gas of χ particles is non-relativistic, which means that $\langle n_\chi(\mathbf{p})\rangle$ is suppressed unless $E_\mathbf{p} \simeq p \ll m_\chi$. Then $\rho \ll \langle \mathcal{L}_{\text{mass}}\rangle$ and $m_{\text{eff}}^2 \ll H^2$.

We note here the special case that the radiation is thermalized with negligible chemical potential. Then

$$\langle n_\chi(\mathbf{p})\rangle = \frac{1}{e^{|\mathbf{p}|/T} - 1}, \tag{A.10}$$

and

$$\langle \mathcal{L}_{\text{kin}}\rangle = \frac{1}{24} m_\chi^2 T^2, \tag{A.11}$$

and using $\rho_\chi = \frac{\pi^2}{30} T^4$,

$$m_{\text{eff}}^2 = \frac{15}{4\pi^2} \left(\lambda_{\text{kin}} - \lambda_{\text{mass}}\right) \Omega_\chi \frac{m_\chi^2}{T^2} H^2 . \tag{A.12}$$

(Notice that the term proportional to λ_{mass} is consistent with the thermal mass-squared given in [1].) In this case, m_{eff}^2/H^2 is of order $\sim m_\chi^2/T^2$ or smaller.

With χ being a fermion, there is no qualitative change in the above results. Taking χ to be a spin $\frac{1}{2}$ fermion, we have

$$\mathcal{L}_{\text{kin}}(\chi) = i\bar{\chi}\, \partial\!\!\!/ \chi, \tag{A.13}$$

$$\mathcal{L}_{\text{mass}}(\chi) = -m_\chi \bar{\chi}\chi . \tag{A.14}$$

Then, the χ field is expanded as

$$\chi(x) = \sum_{\mathbf{p},s} \frac{1}{\sqrt{2E_\mathbf{p} V}} \left(b_{\mathbf{p},s} u_{\mathbf{p},s} e^{-ipx} + d_{\mathbf{p},s}^\dagger v_{\mathbf{p},s} e^{ipx} \right), \tag{A.15}$$

where s is the spin index. Here, $u_{\mathbf{p},s}$ and $v_{\mathbf{p},s}$ are Dirac spinors, and $b_{\mathbf{p},s}$ and $d_{\mathbf{p},s}^\dagger$ are creation and annihilation operators obeying $\{b_{\mathbf{p},s}, b_{\mathbf{p}',s'}^\dagger\} = \{d_{\mathbf{p},s}, d_{\mathbf{p}',s'}^\dagger\} = \delta_{\mathbf{p}\mathbf{p}'}\delta_{s,s'}$. (The following argument does not change even when χ is a Majorana particle.) Dirac equation gives $\langle \mathcal{L}_{\text{kin}}\rangle = -\langle \mathcal{L}_{\text{mass}}\rangle$ and hence, with the lagrangian given in eq. (4.7), we also obtain the same expression of Δm_ϕ^2 as given in eq. (A.6). For a Dirac fermion, expectation value of $\mathcal{L}_{\text{kin}}$ is given by

$$\langle \mathcal{L}_{\text{kin}}\rangle = m_\chi^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\langle n_\chi(\mathbf{p})\rangle + \langle n_{\bar{\chi}}(\mathbf{p})\rangle}{E_\mathbf{p}}, \tag{A.16}$$

where $n_\chi(\mathbf{p}) = \sum_s b_{\mathbf{p},s}^\dagger b_{\mathbf{p},s}$ and $n_{\bar{\chi}}(\mathbf{p}) = \sum_s d_{\mathbf{p},s}^\dagger d_{\mathbf{p},s}$, while the energy density is

$$\rho_\chi = \int \frac{d^3\mathbf{p}}{(2\pi)^3} E_\mathbf{p} \left[\langle n_\chi(\mathbf{p})\rangle + \langle n_{\bar{\chi}}(\mathbf{p})\rangle\right]. \tag{A.17}$$

In the non-relativistic case $E_\mathbf{p}$ in the above expressions can be replaced by m_χ again giving eq. (A.9) except for the factor $\frac{1}{2}$. In addition, in the relativistic case, $\langle \mathcal{L}_{\text{kin}}\rangle = \frac{1}{6} m_\chi^2 T^2$ taking account of the spin degrees of freedom, and hence m_{eff}^2 becomes

$$m_{\text{eff}}^2 = \frac{30}{7\pi^2} \left(\lambda_{\text{kin}} - \lambda_{\text{mass}}\right) \Omega_\chi \frac{m_\chi^2}{T^2} H^2 . \tag{A.18}$$

Thus, the effective mass induced by relativistic fermions is again much smaller than H.

References

[1] L. Dolan and R. Jackiw, *Symmetry behavior at finite temperature*, *Phys. Rev.* **D 9** (1974) 3320.

[2] D.H. Lyth, *Non-renormalizable terms and M-theory during inflation*, *Phys. Lett.* **B 419** (1998) 57 [hep-ph/9710347].

[3] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Supersymmetry breaking and soft terms in M-theory*, *Phys. Lett.* **B 415** (1997) 24 [hep-th/9707143].

[4] B.A. Ovrut and P.J. Steinhardt, *Supersymmetry and inflation: a new approach*, *Phys. Lett.* **B 133** (1983) 161;
G.D. Coughlan, R. Holman, P. Ramond and G.G. Ross, *Supersymmetry and the entropy crisis*, *Phys. Lett.* **B 140** (1984) 44;
M. Dine, W. Fischler and D. Nemeschansky, *Solution of the entropy crisis of supersymmetric theories*, *Phys. Lett.* **B 136** (1984) 169.

[5] E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart and D. Wands, *False vacuum inflation with Einstein gravity*, *Phys. Rev.* **D 49** (1994) 6410 [astro-ph/9401011].

[6] M. Dine, L. Randall and S. Thomas, *Supersymmetry breaking in the early universe*, *Phys. Rev. Lett.* **75** (1995) 398 [hep-ph/9503303]; *Baryogenesis from flat directions of the supersymmetric standard model*, *Nucl. Phys.* **B 458** (1996) 291 [hep-ph/9507453].

[7] E.J. Chun, D. Comelli and D.H. Lyth, *The abundance of relativistic axions in a flaton model of Peccei-Quinn symmetry*, *Phys. Rev.* **D 62** (2000) 095013 [hep-ph/0008133];
E.J. Chun, H.B. Kim and D.H. Lyth, *Cosmological constraints on a Peccei-Quinn flatino as the lightest supersymmetric particle*, *Phys. Rev.* **D 62** (2000) 125001 [hep-ph/0008139].

[8] I. Affleck and M. Dine, *A new mechanism for baryogenesis*, *Nucl. Phys.* **B 249** (1985) 361.

[9] A. Mazumdar and A. Perez-Lorenzana, *Sneutrino bangs*, hep-ph/0311106;
K. Enqvist, S. Kasuya and A. Mazumdar, *MSSM higgses as the source of reheating and all matter*, hep-ph/0311224.

[10] D.H. Lyth and A. Riotto, *Particle physics models of inflation and the cosmological density perturbation*, *Phys. Rept.* **314** (1999) 1 [hep-ph/9807278].

[11] A.R. Liddle and D.H. Lyth, *Cosmological inflation and large-scale structure*, Cambridge University Press 2000.

[12] D.H. Lyth and D. Wands, *Generating the curvature perturbation without an inflaton*, *Phys. Lett.* **B 524** (2002) 5 [hep-ph/0110002].

[13] T. Moroi and T. Takahashi, *Effects of cosmological moduli fields on cosmic microwave background*, *Phys. Lett.* **B 522** (2001) 215 [hep-ph/0110096].

[14] K. Enqvist and M.S. Sloth, *Adiabatic cmb perturbations in pre big bang string cosmology*, *Nucl. Phys.* **B 626** (2002) 395 [hep-ph/0109214].

[15] D.N. Spergel et al., *First year Wilkinson microwave anisotropy probe (wmap) observations: determination of cosmological parameters*, *Astrophys. J. Suppl.* **148** (2003) 175 [astro-ph/0302209].

[16] SDSS collaboration, M. Tegmark et al., *Cosmological parameters from SDSS and WMAP*, astro-ph/0310723.

[17] K. Dimopoulos, G. Lazarides, D. Lyth and R. Ruiz de Austri, *The Peccei-Quinn field as curvaton*, *J. High Energy Phys.* **05** (2003) 057 [hep-ph/0303154].

[18] N. Arkani-Hamed, H.-C. Cheng, P. Creminelli and L. Randall, *Extranatural inflation*, *Phys. Rev. Lett.* **90** (2003) 221302 [hep-th/0301218]; *Pseudonatural inflation*, *JCAP* **07** (2003) 003 [hep-th/0302034].

[19] T. Banks, M. Dine, P.J. Fox and E. Gorbatov, *On the possibility of large axion decay constants*, *JCAP* **06** (2003) 001 [hep-th/0303252].

[20] J. Polchinski, *Superstring theory and beyond*, Cambridge University Press1998.

[21] T. Banks and M. Dine, *Coping with strongly coupled string theory*, *Phys. Rev.* **D 50** (1994) 7454 [hep-th/9406132]; *Couplings and scales in strongly coupled heterotic string theory*, *Nucl. Phys.* **B 479** (1996) 173 [hep-th/9605136].

[22] G. 't Hooft, in *Recent developments in gauge theories*, Plenum Publishing Co. 1980.

[23] K. Dimopoulos and D.H. Lyth, *Models of inflation liberated by the curvaton hypothesis*, hep-ph/0209180;
D.H. Lyth, *Which is the best inflation model?*, hep-th/0311040.

[24] E.D. Stewart, *Inflation, supergravity and superstrings*, *Phys. Rev.* **D 51** (1995) 6847 [hep-ph/9405389].

[25] S. Weinberg, *The quantum theory of fields, vol.* **3***: Supersymmetry*, Cambridge University Press, Cambridge 2000.

[26] M.K. Gaillard and V. Jain, *Supergravity coupled to chiral matter at one loop*, *Phys. Rev.* **D 49** (1994) 1951 [hep-th/9308090].

[27] J. Bagger, E. Poppitz and L. Randall, *Destabilizing divergences in supergravity theories at two loops*, *Nucl. Phys.* **B 455** (1995) 59 [hep-ph/9505244].

[28] J.D. Cohn and E.D. Stewart, *Non-abelian discrete gauge symmetries and inflation*, *Phys. Lett.* **B 475** (2000) 231 [hep-ph/0001333];
E.D. Stewart and J.D. Cohn, *Inflationary models with a flat potential enforced by non-abelian discrete gauge symmetries*, *Phys. Rev.* **D 63** (2001) 083519 [hep-ph/0002214].

[29] E.J. Chun, K. Dimopoulos and D. Lyth, *Curvaton and QCD axion in supersymmetric theories*, hep-ph/0402059.

[30] K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation with pseudo-Nambu-Goldstone bosons*, *Phys. Rev. Lett.* **65** (1990) 3233;
F.C. Adams, J.R. Bond, K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation: particle physics models, power law spectra for large scale structure and constraints from cobe*, *Phys. Rev.* **D 47** (1993) 426 [hep-ph/9207245].

[31] J.D. Cohn and E.D. Stewart, *Non-abelian discrete gauge symmetries and inflation*, *Phys. Lett.* **B 475** (2000) 231 [hep-ph/0001333].

[32] D.E. Kaplan and N.J. Weiner, *Little inflatons and gauge inflation*, *JCAP* **02** (2004) 005 [hep-ph/0302014].

[33] J.P. Hsu, R. Kallosh and S. Prokushkin, *On brane inflation with volume stabilization*, *JCAP* **12** (2003) 009 [hep-th/0311077].

JHEP05(2004)004

[34] J.A. Adams, G.G. Ross and S. Sarkar, *Natural supergravity inflation*, *Phys. Lett.* **B 391** (1997) 271.

[35] K. Dimopoulos, D.H. Lyth, A. Notari and A. Riotto, *The curvaton as a pseudo-Nambu-Goldstone boson*, *J. High Energy Phys.* **07** (2003) 053 [hep-ph/0304050].

[36] F. Koyama, Y. Tachikawa and T. Watari, *Supergravity analysis of hybrid inflation model from D3-D7 system*, hep-th/0311191;
J.P. Hsu and R. Kallosh, *Volume stabilization and the origin of the inflaton shift symmetry in string theory*, hep-th/0402047.

[37] M. Kawasaki, M. Yamaguchi and T. Yanagida, *Natural chaotic inflation in supergravity*, *Phys. Rev. Lett.* **85** (2000) 3572 [hep-ph/0004243];
M. Yamaguchi and J. Yokoyama, *New inflation in supergravity with a chaotic initial condition*, *Phys. Rev.* **D 63** (2001) 043506 [hep-ph/0007021];
M. Yamaguchi, *Natural double inflation in supergravity*, *Phys. Rev.* **D 64** (2001) 063502 [hep-ph/0103045];
M. Yamaguchi and J. Yokoyama, *Chaotic hybrid new inflation in supergravity with a running spectral index*, *Phys. Rev.* **D 68** (2003) 123520 [hep-ph/0307373].

[38] T. Banks and M. Dine, *The cosmology of string theoretic axions*, *Nucl. Phys.* **B 505** (1997) 445 [hep-th/9608197];
T. Banks, M. Dine and M. Graesser, *Supersymmetry, axions and cosmology*, *Phys. Rev.* **D 68** (2003) 075011 [hep-ph/0210256].

[39] M.K. Gaillard, D.H. Lyth and H. Murayama, *Inflation and flat directions in modular invariant superstring effective theories*, *Phys. Rev.* **D 58** (1998) 123505 [hep-th/9806157].

[40] S. Kachru et al., *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [hep-th/0308055].

[41] H. Firouzjahi and S.H.H. Tye, *Closer towards inflation in string theory*, *Phys. Lett.* **B 584** (2004) 147 [hep-th/0312020].

[42] K.A. Olive, *Inflation*, *Phys. Rept.* **190** (1990) 307;
H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, *Chaotic inflation and baryogenesis in supergravity*, *Phys. Rev.* **D 50** (1994) 2356 [hep-ph/9311326].

[43] M.K. Gaillard, H. Murayama and K.A. Olive, *Preserving flat directions during inflation*, *Phys. Lett.* **B 355** (1995) 71 [hep-ph/9504307].

[44] E.D. Stewart, *Flattening the inflaton's potential with quantum corrections*, *Phys. Lett.* **B 391** (1997) 34 [hep-ph/9606241].

[45] E.D. Stewart, *Flattening the inflaton's potential with quantum corrections, 2 Phys. Rev.* **D 56** (1997) 2019 [hep-ph/9703232].

[46] L. Covi, D.H. Lyth and A. Melchiorri, *New constraints on the running-mass inflation model*, *Phys. Rev.* **D 67** (2003) 043507 [hep-ph/0210395].

[47] K. Kadota and E.D. Stewart, *Successful modular cosmology*, *J. High Energy Phys.* **07** (2003) 013 [hep-ph/0304127].

[48] E. Halyo, *Hybrid inflation from supergravity D-terms*, *Phys. Lett.* **B 387** (1996) 43 [hep-ph/9606423].

[49] P. Binetruy and G.R. Dvali, *D-term inflation*, *Phys. Lett.* **B 388** (1996) 241 [hep-ph/9606342].

JHEP05(2004)004

[50] C.F. Kolda and J. March-Russell, *Supersymmetric D-term inflation, reheating and affleck-dine baryogenesis*, *Phys. Rev.* **D 60** (1999) 023504 [hep-ph/9802358].